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#### ABSTRACT

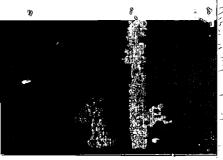
In the reporting of individual student results from standardized tests in educational assessments, the percentile rank of the individual student is a major numerical indicator. For example, in the 1998 and 1999 California Standardized Testing and Reporting (STAR) program using the Stanford Achievement Test Series, Ninth Edition, Form T (Stanford 9), the 1998 "Home Report" and the 1999 "Parent Report" feature solely the National Grade Percentile Ranks. (These percentile rank scores are also featured in the more extensive "Student Report.") This paper develops a formulation and presents calculations to examine the properties of year-1, year-2 comparisons using these individual percentile rank scores. The approach and formulation follow the previous investigations of the accuracy of the individual percentile rank score in D. Rogosa (1999). A typical question that this paper addresses is: What are the chances that a student who really improved 10 percentile points from year-1 (1998) to year-2 (1999) obtains a lower percentile rank in year-2 than year-1? Such questions are addressed using the test reliability coefficient in classical test theory to represent the quality of measurement. These approaches make it possible to investigate the question, What level of test reliability is needed to obtain good accuracy in year-1, year-2 comparisons? (Author/SLD)





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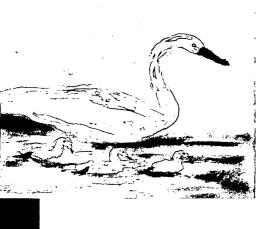




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**CSE Technical Report 510** 

David Rogosa CRESST/Stanford University





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February 2000

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Project 3.3 Validity of Measures of Progress: Measurement of Progress David Director CRESST/Stanford University

David Rogosa, Project

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## Accuracy of Year-1, Year-2 Comparisons Using Individual Percentile Rank Scores: Classical Test Theory Calculations

David Rogosa Stanford University July 1999

### ABSTRACT

In the reporting of individual student results from standardized tests in educational assessments, the percentile rank of the individual student is a major, if not the most prominent, numerical indicator. For example, in the 1998 and 1999 California Standardized Testing and Reporting (STAR) program using the Stanford Achievement Test Series, Ninth Edition, Form T (Stanford 9), the 1998 Home Report and 1999 Parent Report feature solely the National Grade Percentile Ranks. (These percentile rank scores also featured in the more extensive Student Report). This paper develops a formulation and presents calculations to examine the properties of year-1, year-2 comparisons using these individual percentile rank scores. The approach and formulation follows the previous investigations of the accuracy of the individual percentile rank score in Rogosa (1999a). A typical question that this paper addresses is: What are the chances that a student who really improved 10 percentile points from year-1 (1998) to year-2 (1999) obtains a lower percentile rank in year-2 than year-1? Such questions are addressed using the test reliability coefficient in classical test theory to represent quality of measurement. Thus we can investigate the question, What level of test reliability is needed to obtain good accuracy in year-1, year-2 comparisons?



## 1. Technical Formulation

The technical formulation is a basic errors-in-variables model with all components having Gaussian distributions. As in Rogosa (1999a), all that is meant by the phrase "classical test theory calculation" is to identify the calculations as pertaining to the simplest case of constant error variance across the score distribution with continuous, Normally distributed scores. The components of what is referred to as the classical test theory calculation are listed below; use the subscript i=1,2 to indicate properties of year-1 or year-2 measurements.

- The cumulative distribution function of the observed scores Y in the national norming sample is denoted by  $G_i(Y)$  for year-1 or year-2. The classical test theory formulation defines this norming distribution, with density function  $g_i(Y)$ , to be a Normal Distribution; denote the corresponding population mean and standard deviation for Y by  $(\mu_{Ni}, \sigma_{Ni})$  for year-1 or year-2.
- The observed measure Y contains error of measurement  $\epsilon$ . The classical test theory assumptions dictate that the error of measurement, denoted by  $\epsilon$ , has a Normal Distribution with mean 0 and constant variance  $\sigma_{\epsilon i}^2$  across the score distribution: i.e.,  $\epsilon \sim N(0, \sqrt{\sigma_{\epsilon i}^2})$ . (More general formulations, such as  $\sigma_{\epsilon}^2$  depending on the level of the test score, can be incorporated into many of these results, with the overhead of added complexity.)
- The test reliability coefficient is often used as an index of the quality of measurement. The test reliability is defined for the full (norms) population; from the classical test theory formulation, the reliability is  $rel_i = (\sigma_{Ni}^2 \sigma_{\epsilon i}^2)/\sigma_{Ni}^2$ . For a rough, but useful, illustration set the reliability of a 60-item test to be .90 (in line with standardized achievement tests). Then use Spearman-Brown to obtain the rough test length equivalents for various reliability values: reliability .60 .65 .70 .75 .80 .85 .90 .95 number items 10 12 16 20 27 38 60 127
- The norming distributions,  $G_i(Y)$ , are based on fallible Y-scores. An alternative is to consider what the norming distribution would be if measurement had been perfect (i.e., not distorted by error of measurement in Y). At the risk of overcomplicating the notation, denote by  $G_i^*(Y)$  the cumulative distribution function with corresponding mean and standard deviation  $(\mu_{Ni}, [(\sigma_{Ni}^2 \sigma_{\epsilon i}^2)]^{1/2}); G_i^*(Y)$  represents a (hypothetical) norming distribution not distorted by measurement error (i.e., constructed from scores with reliability 1).
- The score for an individual student examinee is denoted by S, The percentile rank (PR) for the score S is 100  $G_i(S)$ ; thus  $G_i(S)$  can be thought of as a nondecreasing transformation of the score S to the percentile rank metric. The score S has underlying true score  $\tau$ ; the measurement model is  $S = \tau + \varepsilon$ . An individual under perfect measurement has percentile rank in observed norming distribution  $100 \ G_i(\tau)$  or, in a norming distribution not distorted by measurement error, the percentile rank is  $100 \ G_i^*(\tau)$ . Often in the calculations, an individual (or an individual's achievement level) is characterized a value of  $G_i^*(\tau)$ .



## 2. Accuracy of Year-to-Year Improvement in Percentile Rank Scores

The observed improvement is  $G_2(S_2) - G_1(S_1)$ , the signed difference between the percentile rank scores for year-2 and year-1; improvement may be positive or negative. The main accuracy calculation is for the quantity:

$$y1y2 = Pr\{G_2(S_2) - G_1(S_1) \le bound \mid G_1^*(\tau_1), G_2^*(\tau_2)\}$$
, (1)

the probability that the improvement in the percentile rank scores is less than or equal to the quantity "bound" (bound may be negative or positive) for a student with stated year-1 and year-2 values  $G_1^*(\tau_1)$  and  $G_2^*(\tau_2)$ .

## 2.1 Computation of y1y2 Probability: Technical Details

Preliminaries. Let  $\Phi[\mathbf{x}]$  indicate the distribution function (cdf) for N(0,1) and  $\Phi[\mathbf{x}]$  indicate the density (pdf) for N(0,1). Then  $G_i(\mathbf{x}) = \Phi[(\mathbf{x} - \mu_{Ni})/\sigma_{Ni}]$ , and  $S_i \mid \tau_i \sim N[\tau_i, \sigma_{Ni}(1 - \mathrm{rel}_i)^{\frac{1}{2}}]$  so that  $\Pr\{S_i \leq \mathbf{x}\} = \Phi[(\mathbf{x} - \tau_i)/\sigma_{Ni}(1 - \mathrm{rel}_i)^{\frac{1}{2}}]$ . Also note that  $\tau_i = \mu_{Ni} + \sigma_{Ni}$  ( $\sqrt{\mathrm{rel}_i}$ )  $\Phi^{-1}[G_i^*(\tau)]$  and  $G_i^{-1}[G_i^*(\tau) + p] = \mu_{Ni} + \sigma_{Ni}$   $\Phi^{-1}[G_i^*(\tau) + p]$ .

The computation of the y1y2 probability is implemented using the following conditioning argument. For a student having a specified value for  $G_1^*(\tau_1)$ , condition on a draw of an  $s_1$  from the  $S_1$ -distribution ( $S_1 | \tau_1 \sim N[\tau_1, \sigma_{N1}(1 - rel_1)^2]$ ) and express that  $S_1$ -value in terms of its fractile of the  $S_1$ -distribution,  $ps_1$ , to obtain:

$$\Pr\{G_2(S_2) - G_1(S_1) \le bound \mid ps_1\} = \Pr\{S_2 \le G_2^{-1}[G_1(S_1) + bound] \mid ps_1\} = \Pr\{G_2(S_2) - G_1(S_1) \le bound \mid ps_1\} = \Pr\{S_2 \le G_2^{-1}[G_1(S_1) + bound] \mid ps_1\} = \Pr\{G_2(S_2) - G_1(S_1) \le bound \mid ps_1\} = \Pr\{G_2(S_2) - G_1(S_1) \le bound \mid ps_1\} = \Pr\{G_2(S_2) - G_1(S_1) \le bound \mid ps_1\} = \Pr\{G_2(S_2) - G_1(S_1) + bound\} = \Pr\{G_2(S_2) - G_1(S_2) + bound\} = \Pr\{G_2(S_2) - G_1(S_2) + bound\} = \Pr\{G_2(S_2) - G_1(S_2) + bound\} = \Pr\{G_2(S_2) - G_2(S_2) + bound\} = \Pr\{G_2$$

$$\Phi \left[ \left\{ \Phi^{-1} \left[ \Phi \left[ (1 - \text{rel}_{1})^{\frac{1}{2}} \Phi^{-1} \left[ \text{ps}_{1} \right] + (\sqrt{\text{rel}_{1}}) \Phi^{-1} \left[ G_{1}^{*} (\tau_{1}) \right] \right] + \text{bound} \right] - (\sqrt{\text{rel}_{2}}) \Phi^{-1} \left[ G_{2}^{*} (\tau_{2}) \right] \right\} / (1 - \text{rel}_{2})^{\frac{1}{2}} \right] .$$
(2)

As a side note to (2),  $G_1(S_1)$  can be expressed in terms of  $ps_1$  as:

$$G_1(S_1) = \Phi[(1 - rel_1)^{\frac{1}{2}} \Phi^{-1}[ps_1] + (\sqrt{rel_1}) \Phi^{-1}[G_1^{\star}(\tau_1)]] .$$

Then uncondition (2) by integrating  $\Pr\{G_2(S_2) - G_1(S_1) \le \text{bound} \mid ps_1\}$  over  $ps_1$  in [0,1]:

$$y1y2 = \int_{0}^{1} \left[ \Phi \left[ \left\{ \Phi^{-1} \left[ \Phi \left[ (1 - \text{rel}_{1})^{\frac{1}{2}} \Phi^{-1} \left[ \text{ps}_{1} \right] + (\sqrt{\text{rel}_{1}}) \Phi^{-1} \left[ G_{1}^{\star} (\tau_{1}) \right] \right] + \text{bound} \right] - (\sqrt{\text{rel}_{2}}) \Phi^{-1} \left[ G_{2}^{\star} (\tau_{2}) \right] \right\} / (1 - \text{rel}_{2})^{\frac{1}{2}} \right] dps_{1}$$
 (3)



## 2.2 Calculations and Illustrations

Maintaining Percentile Rank,  $G_1^*(\tau_1) = G_2^*(\tau_2)$ . Table 1 displays values of y1y2 in (3) for a student who has maintained percentile rank in year-1 and year-2 in the sense of  $G_1^*(\tau_1)$  is set equal to  $G_2^*(\tau_2)$ . The entries in Table 1 also use the simplification of (3) in setting the year-1 and year-2 test reliability coefficients to be equal, rel<sub>1</sub> = rel<sub>2</sub>. Consequences of different reliabilities are also discussed below.

Table 1 presents values of y1y2 in (3) for test reliability values from .70 to .95 (.7, .8, .85, .9, .925, .95). For each reliability value the rows of each sub-table represent a student's value of  $G_1^*(\tau_1) = G_2^*(\tau_2)$ , so that the .60 row indicates a student who "really belongs" at the  $60^{th}$  percentile in both year-1 and year-2. For a test with reliability .90 administered in year-1 and year-2, that student has a 11.6% chance of showing a decrease of at least 20 percentile points and also a 11.6% chance of showing an increase of at least 20 percentile points.

To compare these accuracy results for year-1, year-2 comparisons in Table 1 with more traditional assessments of uncertainty, consider values of the standard error of  $G_2(S_2)-G_1(S_1)$ . The results for s.e.[ $G_2(S_2)-G_1(S_1)$ ] are obtained from derivations of the moments of the percentile rank score in Rogosa (1999c). For both year-1 and year-2 tests having reliability .90, s.e.[ $G_2(S_2)-G_1(S_1)$ ] is 0.1702 for  $G_1^*(\tau_1)=G_2^*(\tau_2)=$ .50. Increase the test reliabilities to .95 and this s.e.[ $G_2(S_2)-G_1(S_1)$ ] becomes 0.1231.

### Insert Table 1 here

Another version of the statement about the student with  $G_1^{\star}(\tau_1) = G_2^{\star}(\tau_2)$ , = .60 and test reliability .90 is that the probability is .768 that the magnitude of the observed change,  $|G_2(S_2) - G_1(S_1)|$ , is less than .20. Figure 1 displays these type of probability statements by plotting the values of  $Pr\{G_2(S_2) - G_1(S_1) \leq \text{bound}\} - Pr\{G_2(S_2) - G_1(S_1) \leq -\text{bound}\}$  as a function of test reliability for  $G_1^{\star}(\tau_1) = G_2^{\star}(\tau_2) = .50$  and .75 or .25.

### Insert Figure 1 here

The result for y1y2 in (3) allows the year-1 and year-2 test reliability coefficients, rel<sub>1</sub> and rel<sub>2</sub> to differ. The consequences of different test reliabilities can be charted in various ways. Rather than be exhaustive, some specific examples are considered here. With  $G_1^*(\tau_1) = G_2^*(\tau_2) = .50$ , the effect of differing test reliabilities is minimal in the following set-up. Taking the base as rel<sub>1</sub> = rel<sub>2</sub> = .90, the difference in y1y2 values between the base (i.e., Table 1 entries) and differing reliabilities rel<sub>1</sub> = .85, rel<sub>2</sub> = .95 is less than .001 for the listed values of bound. The same result is found for comparing rel<sub>1</sub> = rel<sub>2</sub> = .80 with rel<sub>1</sub> = .75, rel<sub>2</sub> = .85. Moving away from  $G_1^*(\tau_1) = G_2^*(\tau_2) = .50$ , differences do emerge. Take  $G_1^*(\tau_1) = G_2^*(\tau_2) = .75$ , then the



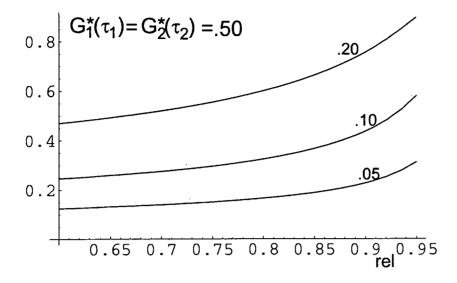
 $\begin{array}{l} \text{Table 1.} \\ \Pr\{G_2(S_2) - G_1(S_1) \leq \text{bound } \mid G_1^{\star}(\tau_1), \ G_2^{\star}(\tau_2)\} \ \text{for reliability .70 to .95 and } G_1^{\star}(\tau_1) = \ G_2^{\star}(\tau_2). \end{array}$ 

Reliability .70													
	20	15	10	05	0.0	.05	.10	.15	. 2				
$G_1^{\star}(\tau_1)$	•												
.10	0.124	0.183	0.263	0.369	0.5	0.631	0.737	0.817	0.876				
.20	0.186	0.248	0.323	0.408	0.5	0.592	0.677	0.752	0.814				
.30	0.219	0.279	0.348	0.422	0.5	0.578	0.652	0.721	0.781				
.40	0.235	0.294	0.359	0.428	0.5	0.572	0.641	0.706	0.765				
.50	0.24	0.299	0.363	0.43	0.5	0.57	0.637	0.701	0.76				
.60	0.235	0.294	0.359	0.428	0.5	0.572	0.641	0.706 0.721	0.765 0.781				
.70	0.219	0.279	0.348	0.422	0.5	0.578 0.592	0.652 0.677	0.721	0.781				
.80	0.186	0.248	0.323 0.263	0.408 0.369	0.5 0.5	0.631	0.877	0.732	0.876				
. 90	0.124	0.183	0.263	0.303	0.5	0.031	0.757	0.027	0.070				
Reliabili	tv .80				•								
	3	bo	und										
	20	15	10	05	0.0	.05	.10	.15	. 2				
$G_1^{\star}(\tau_1)$													
.10	0.072	0.126	0.211	0.337	0.5	0.663	0.789	0.874	0.928				
.20	0.137	0.202	0.287	0.388	0.5	0.612	0.713	0.798	0.863				
.30	0.174	0.241	0.319	0.407	0.5	0.593	0.681	0.759	0.826				
.40	0.194	0.259	0.333	0.415	0.5	0.585	0.667	0.741	0.806				
.50	0.2	0.265	0.338	0.417	0.5	0.583	0.662 0.667	0.735 0.741	0.8 0.806				
.60	0.194	0.259	0.333	0.415 0.407	0.5 0.5	0.585 0.593	0.681	0.759	0.826				
.70	0.174 0.137	0.241 0.202	0.319 0.287	0.388	0.5	0.612	0.713	0.798	0.863				
.80 .90	0.137	0.126	0.211	0.337	0.5	0.663	0.789	0.874	0.928				
Reliabili	ty .85	bo	ound										
Reliabili	•	bc 15	ound 10	05	0.0	. 05	.10	.15	. 2				
	ty .85			05	0.0	. 05	.10	.15	. 2				
$G_1^{\star}(\tau_1)$	20			05 0.311	0.0	.05 0.689	.10 0.827	.15 0.911	.2 0.957				
	•	15	10	0.311 0.371			0.827 0.742	0.911 0.832	0.957 0.897				
$G_1^{\star}(\tau_1)$	20 0.043	15 0.089	10 0.173 0.258 0.295	0.311 0.371 0.394	0.5 0.5 0.5	0.689 0.629 0.606	0.827 0.742 0.705	0.911 0.832 0.79	0.957 0.897 0.859				
$G_1^*(\tau_1)$ .10 .20 .30 .40	20 0.043 0.103 0'.141 0.162	15 0.089 0.168 0.21 0.23	10 0.173 0.258 0.295 0.312	0.311 0.371 0.394 0.403	0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597	0.827 0.742 0.705 0.688	0.911 0.832 0.79 0.77	0.957 0.897 0.859 0.838				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ \end{array}$	20 0.043 0.103 0'.141 0.162 0.168	15 0.089 0.168 0.21 0.23 0.237	10 0.173 0.258 0.295 0.312 0.317	0.311 0.371 0.394 0.403 0.406	0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594	0.827 0.742 0.705 0.688 0.683	0.911 0.832 0.79 0.77 0.763	0.957 0.897 0.859 0.838 0.832				
$\begin{array}{c} G_{l}^{\star}(\tau_{l}) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \end{array}$	20 0.043 0.103 0'.141 0.162 0.168 0.162	15 0.089 0.168 0.21 0.23 0.237 0.23	10 0.173 0.258 0.295 0.312 0.317 0.312	0.311 0.371 0.394 0.403 0.406 0.403	0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597	0.827 0.742 0.705 0.688 0.683 0.688	0.911 0.832 0.79 0.77 0.763	0.957 0.897 0.859 0.838 0.832 0.838				
$\begin{array}{c} G_{l}^{\star}(\tau_{l}) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ \end{array}$	20 0.043 0.103 0'.141 0.162 0.168 0.162 0.141	15 0.089 0.168 0.21 0.23 0.237 0.23	10 0.173 0.258 0.295 0.312 0.317 0.312 0.295	0.311 0.371 0.394 0.403 0.406 0.403 0.394	0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606	0.827 0.742 0.705 0.688 0.683 0.688 0.705	0.911 0.832 0.79 0.77 0.763 0.77	0.957 0.897 0.859 0.838 0.832 0.838 0.859				
$\begin{array}{c} G_{l}^{\star}(\tau_{l}) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \end{array}$	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21	10 0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371	0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629	0.827 0.742 0.705 0.688 0.683 0.688 0.705	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832	0.957 0.897 0.859 0.838 0.832 0.838				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \end{array}$	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043	15 0.089 0.168 0.21 0.23 0.237 0.23	10 0.173 0.258 0.295 0.312 0.317 0.312 0.295	0.311 0.371 0.394 0.403 0.406 0.403 0.394	0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606	0.827 0.742 0.705 0.688 0.683 0.688 0.705	0.911 0.832 0.79 0.77 0.763 0.77	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897				
$\begin{array}{c} G_{l}^{\star}(\tau_{l}) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \end{array}$	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089	10 0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371	0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629	0.827 0.742 0.705 0.688 0.683 0.688 0.705	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897				
G <sub>1</sub> *(τ <sub>1</sub> ) .10 .20 .30 .40 .50 .60 .70 .80 .90	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089	10 0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371	0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629	0.827 0.742 0.705 0.688 0.683 0.688 0.705	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897				
$G_{l}^{\star}(\tau_{1})$ .10 .20 .30 .40 .50 .60 .70 .80 .90  Reliabili	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.683 0.688 0.705 0.742 0.827	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \end{array}$ Reliabili	20 0.043 0.103 0'.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  pund10 0.121	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.683 0.688 0.705 0.742 0.827	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$G_{1}^{\star}(\tau_{1})$ .10 .20 .30 .40 .50 .60 .70 .80 .90  Reliabili $G_{1}^{\star}(\tau_{1})$ .10 .20	20 0.043 0.103 0'.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  pund10  0.121 0.213	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.683 0.688 0.705 0.742 0.827	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.859 0.957				
$\begin{array}{c} G_l^{\star}(\tau_l) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \end{array}$ Reliabili	20 0.043 0.103 0'.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06 0.096	15 0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089 bc 15 0.047 0.119 0.163	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  ound10  0.121 0.213 0.256	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.683 0.688 0.705 0.742 0.827	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_l^{\star}(\tau_l) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \end{array}$ Reliabili	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06 0.096 0.116	15  0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089  bc15  0.047 0.119 0.163 0.186	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  ound10  0.121 0.213 0.256 0.276	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.688 0.705 0.742 0.827	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911 .15 0.953 0.881 0.837	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_{l}^{\star}(\tau_{l}) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06 0.096 0.116 0.123	15  0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089  bc15  0.047 0.119 0.163 0.186 0.193	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  ound10  0.121 0.213 0.256 0.276 0.282	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311 05	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.688 0.705 0.742 0.827 .10 0.879 0.787 0.744 0.724	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911 .15 0.953 0.881 0.837 0.814	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	20  0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .90 20  0.017 0.06 0.096 0.116 0.123 0.116	15  0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089  bc15  0.047 0.119 0.163 0.186 0.193 0.186	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  ound10  0.121 0.213 0.256 0.276 0.282 0.276	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.688 0.705 0.742 0.827 .10 0.879 0.787 0.744 0.724 0.718	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911 .15 0.953 0.881 0.837 0.814 0.807	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \end{array}$ Reliabili $\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ \end{array}$	20 0.043 0.103 0.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06 0.096 0.116 0.123	15  0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089  bc15  0.047 0.119 0.163 0.186 0.193	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  ound10  0.121 0.213 0.256 0.276 0.282	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311 05 0.27 0.344 0.372 0.383 0.387 0.383	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.699 0.629 0.689	0.827 0.742 0.705 0.688 0.688 0.705 0.742 0.827 .10 0.879 0.787 0.744 0.724 0.724 0.724 0.724 0.724	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911 .15 0.953 0.881 0.837 0.814 0.807 0.814	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				
$\begin{array}{c} G_1^{\star}(\tau_1) \\ .10 \\ .20 \\ .30 \\ .40 \\ .50 \\ .60 \\ .70 \\ .80 \\ .90 \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\$	20 0.043 0.103 0'.141 0.162 0.168 0.162 0.141 0.103 0.043  ty .9020 0.017 0.06 0.096 0.116 0.123 0.116 0.096	15  0.089 0.168 0.21 0.23 0.237 0.23 0.21 0.168 0.089  bc15  0.047 0.119 0.163 0.186 0.193 0.186 0.163	10  0.173 0.258 0.295 0.312 0.317 0.312 0.295 0.258 0.173  pund10  0.121 0.213 0.256 0.276 0.282 0.276 0.256	0.311 0.371 0.394 0.403 0.406 0.403 0.394 0.371 0.311 05 0.27 0.344 0.372 0.383 0.387 0.383	0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5	0.689 0.629 0.606 0.597 0.594 0.597 0.606 0.629 0.689	0.827 0.742 0.705 0.688 0.683 0.688 0.705 0.742 0.827 .10 0.879 0.787 0.744 0.724 0.718 0.724 0.724	0.911 0.832 0.79 0.77 0.763 0.77 0.79 0.832 0.911 .15 0.953 0.881 0.837 0.814 0.807 0.814 0.837	0.957 0.897 0.859 0.838 0.832 0.838 0.859 0.897 0.957				



Reliab	ility .925								
	J	bo	ound						
	20	15	10	05	0.0	.05	.10	.15	. 2
$G_1^*(\tau_1)$									
.10	0.007	0.026	0.087	0.238	0.5	0.762	0.913	0.974	0.993
.20	0.037	0.087	0.18	0.322	0.5	0.678	0.82	0.913	0.963
.30	0.067	0.13	0.226	0.353	0.5	0.647	0.774	0.87	0.933
.40	0.085	0.153	0.248	0.367	0.5	0.633	0.752	0.847	0.915
.50	0.091	0.16	0.254	0.371	0.5	0.629	0.746	0.84	0.909
.60	0.085	0.153	0.248	0.367	0.5	0.633	0.752	0.847	0.915
.70	0.067	0.13	0.226	0.353	0.5	0.647	0.774	0.87	0.933
.80	0.037	0.087	0.18	0.322	0.5	0.678	0.82	0.913	0.963
.90	0.007	0.026	0.087	0.238	0.5	0.762	0.913	0.974	0.993
Reliah	ility .95								
1001140	11109 .00	bo	ound						
	20	15	10	05	0.0	.05	.10	.15	.2
$G_1^*(\tau_1)$									
.10	0.001	0.008	0.047	0.19	0.5	0.81	0.953	0.992	0.999
.20	0.014	0.048	0.131	0.286	0.5	0.714	0.869	0.952	0.986
.30	0.033	0.085	0.18	0.323	0.5	0.677	0.82	0.915	0.967
.40	0.047	0.106	0.203	0.339	0.5	0.661	0.797	0.894	0.953
.50	0.052	0.113	0.21	0.344	0.5	0.656	0.79	0.887	0.948
.60	0.047	0.106	0.203	0.339	0.5	0.661	0.797	0.894	0.953
.70	0.033	0.085	0.18	0.323	0.5	0.677	0.82	0.915	0.967
.80	0.014	0.048	0.131	0.286	0.5	0.714	0.869	0.952	0.986
9.0	0 001	0.008	0.047	0.19	0.5	0.81	0.953	0.992	0.999





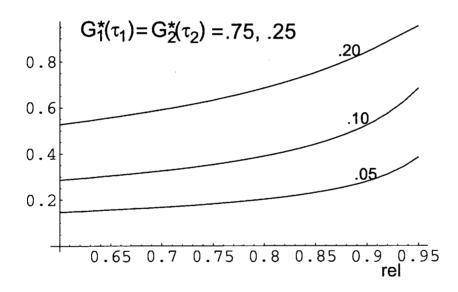


Figure 1. Plots of  $\Pr\{G_2(S_2)-G_1(S_1)\leq \text{bound}\}-\Pr\{G_2(S_2)-G_1(S_1)\leq -\text{bound}\}$  as a function of test reliability for  $G_1^{\star}(\tau_1)=G_2^{\star}(\tau_2)=.50$  in top frame and  $G_1^{\star}(\tau_1)=G_2^{\star}(\tau_2)=.75$  or .25 in bottom frame for labeled values of bound = {.05, .10, .20}.



differences in y1y2 values for rel $_1$  = rel $_2$  = .80 are larger by .02 to .03 than y1y2 values for rel $_1$  = .75, rel $_2$  = .85 and smaller by .02 to .03 than y1y2 values for rel $_1$  = .85, rel $_2$  = .75. But the two-sided probability statement used in Figure 1, Pr{  $G_2(S_2) - G_1(S_1) \le DOUND}$  changes less than .002 for bound = .10 and with rel $_1$  = rel $_2$  = .80 compared to rel $_1$  = .9, rel $_2$  = .7 or rel $_1$  = .7, rel $_2$  = .9. With bound = .20, the change in y1y2 values is less than .001 for these different reliability configurations. Calculations for year-1, year-2 comparisons based on an actual standardized achievement test, Stanford 9, and in which the tests have different reliabilities (and different norms distributions) can be found in Rogosa (1999b).

Increasing Percentile Rank,  $G_1^*(\tau_1) + .10 = G_2^*(\tau_2)$ . Table 2 displays values of y1y2 in (3) for a student who improved percentile rank 10 points from year-1 to year-2 in the sense of  $G_2^*(\tau_2) = G_1^*(\tau_1) + .10$ . The entries in Table 2 also use the simplification of (3) in setting the year-1 and year-2 test reliability coefficients to be equal. One basic question Table 2 informs about is, What's the probability of seeing a decline in the observed percentile rank, even when the student has made a noticeable improvement? (by setting bound = 0 in Table 2). For  $G_1^*(\tau_1) = .4$  or .5, that probability is .36 for test reliability .8, .295 for test reliability .9 and .217 for test reliability .95.

#### Insert Table 2 here

These probability statements,  $\Pr\{G_2(S_2)-G_1(S_1)\leq \text{bound}\}$ , such as in Table 2 allow a decline in scores to result from  $G_1(S_1)$  being "too high"  $(G_1(S_1)>G_1^*(\tau_1))$  as much from  $G_2(S_2)$  being much "too low." Another view of these kind of calculations can be obtained from Equation (2) by setting  $ps_1=.5$ , which results in a fixing of the value of  $G_1(S_1)=G_1(\tau_1)=\Phi[\sqrt{rel_1})\Phi^{-1}[G_1^*(\tau_1)]$ ]. Thus  $G_2(S_2)$  is the only random component in the student improvement. And for the simplest comparison take  $G_1^*(\tau_1)=.50$ , as that results in  $G_1(S_1)=G_1(\tau_1)=G_1^*(\tau_1)=.50$ . By fixing  $ps_1=.5$  a large random component of  $G_2(S_2)-G_1(S_1)$  is removed, and thus we would expect a quantity such as  $\Pr\{G_2(S_2)-G_1(S_1)\leq 0\}$  given a "true" increase  $G_2^*(\tau_2)=G_1^*(\tau_1)+.10$  would become smaller than the results shown in Table 2 (which are obtained from Equation 3). Below is a comparison of  $\Pr\{G_2(S_2)-G_1(S_1)\leq 0\}$  given  $G_2^*(\tau_2)=.6$ ,  $G_1^*(\tau_1)=.5$  from Equation (2) (fixing  $G_1(S_1)=.5$ ) and Equation (3). The Equation (2) quantities are smaller, but perhaps not by as much as expected (especially for the lower reliability values).

```
\begin{array}{llll} & & Pr\{\,G_2(S_2)-G_1(S_1)\,\leq\,0\},\, given\,\,G_2^{\star}(\tau_2)=.6,\,G_1^{\star}(\tau_1)=.5\\ \text{rel} & 0.8 & 0.825 & 0.85 & 0.875 & 0.9 & 0.925 & 0.95\\ \text{eq2} & 0.306 & 0.291 & 0.273 & 0.251 & 0.224 & 0.187 & 0.135\\ \text{eq3} & 0.360 & 0.349 & 0.335 & 0.318 & 0.295 & 0.265 & 0.217 \end{array}
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Table 2.  $\Pr\{G_2(S_2) - G_1(S_1) \leq \text{bound} \mid G_1^{\star}(\tau_1), \ G_2^{\star}(\tau_2)\}\$  for reliability .80 to .95 and  $G_1^{\star}(\tau_1) + .10 = \ G_2^{\star}(\tau_2)$ .

		.30		.88	.85	.83	. 82	.82	0.834	. 85	. 88			.30		. 95	. 92	. 90	.89	0.892	90	. 92	. 95			.30		. 99	.97	96.	0.956	. 95	96.	.97	. 99
		.25		ω.	. 79	۲.	. 7	۲.	77.0	۲.	. 82			.25		. 90	.86	.83	. 82	0.825	.83	. 86	.90			. 25		. 97	. 93	.91	0.899	.89	. 91	. 93	ο.
		7.		. 74	. 71	. 69	.68	.68	0.696	. 71	. 74					. 82	.77	. 74	. 73	0.737	. 74	.77	. 82			. 2		90	. 84	. 81	0.803	. 80	.81	. 84	. 90
		.15		.64	. 62	.61	.60	. 60		.62	. 64			.15		9.	. 65	. 63	. 63	0.632	.63	. 65	9.			.15		۲.	. 70	9.	9.	9.	9.	.70	0.752
		.10		. 52	. 52	. 52	. 52	. 52	0.526	. 52	. 52			.10		.51	.51	.51	.51	0.517	. 51	. 51	.51			.10		. 51	. 51	. 51	. 51	. 51	. 51	. 51	0.511
		. 05		.39	4.	.43	4.	.44	43	4.	. 39			. 05		ε.	.37	.39	.40	0.402	.39	.37	ω.			. 05		. 25	.31	.34	.35	.35	.34	.31	0.256
		0.0	-	. 26	ω.	.35	•	ъ.	0.351	ω.	. 26			0.0		Τ.		. 28	.29	0.295	. 28	7	۲.			0.0		. 08	.16	.20	.21	.21	. 20	.16	0
		05		.16	. 24	.27	•	. 28	0.272	•	.16			05		.07	. 15	. 18	.20	0.204	. 18	. 15	.07			05		٥.	٥.	٦.	۲.	٦.	۲.	٥.	0.021
	ponnoq	10		۲.	۲.	.20	.21	.21	0.203	0.17	۲.		ound	10		0.	٥.	۲.	٦.	0.132	٦.	٥.	٥.		ponnoq	10		00.	. 02	. 04	•	. 05	.04	. 02	00
	рq	15		0.058	0.114	0.146	0.161	0.161	0.146	0.114	0.058		Ā	15		0.011	0.042	0.067	0.079	0.079	0.067	0.042	0.011		ğ	15		0.001	0.007	0.017	0.023	0.023	0.017	0.007	0.001
lity .80		20		0.032	0.074	0.101	0.114	0.114	0.101	0.074	0.032	lity .90		20		0.004	0.019	0.035	0.044	0.044	0.035	0.019	0.004	lity .95		20		0.	0.002	0.005	0.008	0.008	0.005	0.002	0
Reliability .80			$G_1^{ullet}( au_1)$	.10	.20	.30	.40	.50	. 60	. 70	.80	Reliability .90			$G_{\mathbf{t}}^{\star}(\tau_{\mathbf{t}})$	.10	.20	.30	.40	.50	.60	.70	.80	Reliability			$G_1^{\star}(\tau_1)$	10	.20	.30	.40	.50	. 60	. 70	. 80

Increasing Percentile Rank,  $G_1^*(\tau_1) + .20 = G_2^*(\tau_2)$ . Table 3 displays values of y1y2 in (3) for an even larger student improvement from year-1 to year-2 in the sense of  $G_2^*(\tau_2) = G_1^*(\tau_1) + .20$ . The entries in Table 3 also use the simplification of (3) in setting the year-1 and year-2 test reliability coefficients to be equal. Again, one question to examine is, What's the probability of seeing a decline in the observed percentile rank, even when the student has made a noticeable improvement? Setting bound = 0 in Table 3, for  $G_1^*(\tau_1) = .3$ , that probability is .229 for test reliability .8, .133 for test reliability .9 and .053 for test reliability .95.

Insert Table 3 here



Table 3.  $\Pr\{G_2(S_2) - G_1(S_1) \le \text{bound} \mid G_1^{\star}(\tau_1), \ G_2^{\star}(\tau_2)\}\ \text{for reliability .80 to .95 and } G_1^{\star}(\tau_1) + .20 = \ G_2^{\star}(\tau_2)\ .$ 

0.142 0.223 0.221 0.32 0.4431 0.543 0.648 0.741 0.818 0.878 0.1442 0.203 0.277 0.362 0.4455 0.555 0.642 0.726 0.8 0.8 0.861 0.142 0.229 0.239 0.378 0.466 0.551 0.636 0.717 0.789 0.85 0.142 0.229 0.239 0.378 0.465 0.551 0.636 0.717 0.789 0.85 0.142 0.229 0.378 0.465 0.551 0.636 0.717 0.789 0.861 0.142 0.229 0.277 0.362 0.455 0.551 0.636 0.717 0.789 0.85 0.142 0.203 0.277 0.362 0.455 0.551 0.648 0.714 0.786 0.861 0.805 0.142 0.229 0.378 0.465 0.551 0.636 0.717 0.789 0.85 0.861 0.086 0.142 0.229 0.378 0.465 0.551 0.648 0.714 0.789 0.85 0.861 0.080 0.142 0.227 0.312 0.245 0.551 0.648 0.776 0.889 0.861 0.805 0.005 0.18 0.28 0.405 0.555 0.653 0.776 0.889 0.945 0.005 0.180 0.220 0.054 0.113 0.228 0.418 0.255 0.653 0.776 0.884 0.912 0.005 0.133 0.208 0.309 0.418 0.535 0.654 0.76 0.848 0.912 0.005 0.133 0.208 0.309 0.418 0.535 0.654 0.76 0.889 0.945 0.005 0.133 0.208 0.309 0.418 0.535 0.654 0.76 0.884 0.925 0.005 0.005 0.130 0.227 0.131 0.235 0.684 0.802 0.889 0.945 0.005 0.133 0.208 0.309 0.418 0.235 0.684 0.802 0.889 0.945 0.005 0.105	Reliability .80	05	bound 0.0	.05	.10	.15	.20	. 25	.30	. 35	. 40
bound  bound  colors	0.08	96	0.142		ω.	4.	•	. 64	•	.81	ω.
bound  bound  0.029  0.299  0.378  0.463  0.551  0.636  0.717  0.789  0.0237  0.306  0.383  0.466  0.551  0.634  0.714  0.785  0.0237  0.306  0.383  0.465  0.551  0.636  0.717  0.789  0.0203  0.277  0.362  0.455  0.55  0.642  0.726  0.889  0.0142  0.221  0.32  0.431  0.543  0.648  0.741  0.818  bound  0.0  0.54  0.119  0.227  0.371  0.53  0.68  0.802  0.864  0.0133  0.208  0.303  0.415  0.535  0.654  0.76  0.848  0.0134  0.208  0.303  0.415  0.535  0.654  0.76  0.848  0.0135  0.208  0.303  0.415  0.535  0.654  0.76  0.848  0.0136  0.18  0.28  0.402  0.535  0.654  0.76  0.848  0.0137  0.208  0.303  0.415  0.535  0.654  0.76  0.848  0.0138  0.208  0.303  0.415  0.535  0.654  0.76  0.848  0.0139  0.139  0.227  0.371  0.535  0.68  0.802  0.889  0.0139  0.042  0.131  0.3  0.544  0.703  0.843  0.916  0.0054  0.119  0.227  0.371  0.53  0.68  0.802  0.899  0.0055  0.120  0.227  0.371  0.53  0.68  0.802  0.955  0.005  0.121  0.217  0.358  0.524  0.688  0.824  0.916  0.005  0.121  0.225  0.363  0.524  0.688  0.824  0.916  0.005  0.121  0.225  0.341  0.524  0.688  0.824  0.916  0.005  0.191  0.341  0.358  0.524  0.688  0.802  0.916  0.005  0.005  0.19  0.341  0.354  0.688  0.802  0.916  0.005  0.005  0.19  0.341  0.524  0.688  0.802  0.916  0.005  0.005  0.19  0.341  0.524  0.688  0.802  0.916  0.005  0.005  0.19  0.341  0.358  0.524  0.688  0.802  0.916  0.005  0.005  0.19  0.341  0.358  0.524  0.688  0.802  0.916		2	0.203	•	ω.	4.	•	. 64	•	∞.	0.861
bound  0.0237 0.306 0.383 0.466 0.551 0.634 0.714 0.785  0.0229 0.299 0.378 0.463 0.551 0.636 0.717 0.789  0.0229 0.299 0.378 0.463 0.551 0.636 0.717 0.789  0.0231 0.221 0.32 0.431 0.543 0.648 0.741 0.818  bound  0.0 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889  0.0133 0.208 0.303 0.415 0.535 0.653 0.776 0.848  0.0133 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.0134 0.216 0.309 0.418 0.535 0.654 0.76 0.848  0.013 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.013 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.013 0.208 0.309 0.415 0.535 0.654 0.76 0.889  0.013 0.208 0.309 0.415 0.535 0.654 0.76 0.889  0.0054 0.119 0.227 0.371 0.53 0.68 0.802 0.809  0.0059 0.120 0.301 0.301 0.301 0.301 0.501 0.703 0.841 0.916  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  0.0053 0.115 0.227 0.341 0.524 0.688 0.824 0.916  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  0.0053 0.0053 0.19 0.341 0.524 0.608 0.824 0.916  0.0053 0.0054 0.19 0.341 0.524 0.608 0.824 0.916  0.0053 0.0054 0.19 0.341 0.524 0.608 0.824 0.916	0.169	6	0.229	•	ω.	4.	•	. 63	•	. 78	0.85
bound  0.0229 0.299 0.378 0.463 0.551 0.636 0.717 0.789  0.0203 0.277 0.362 0.455 0.55 0.642 0.726 0.8  0.142 0.221 0.32 0.431 0.543 0.648 0.741 0.818  bound  0.0 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889  0.133 0.208 0.309 0.418 0.535 0.663 0.776 0.849  0.133 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.016 0.18 0.28 0.402 0.535 0.654 0.76 0.848  0.013 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.013 0.208 0.303 0.415 0.535 0.654 0.76 0.848  0.013 0.208 0.303 0.415 0.535 0.653 0.776 0.848  0.010 0.028 0.301 0.415 0.535 0.663 0.776 0.848  0.0094 0.119 0.227 0.371 0.53 0.68 0.802 0.889  10 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.899  10 0.055 0.015 0.015 0.341 0.524 0.703 0.843 0.916  11 0.053 0.123 0.225 0.358 0.524 0.688 0.824 0.916  12 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  13 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  14 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  15 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916  16 0.055 0.015 0.089 0.19 0.341 0.524 0.703 0.843 0.931  17 0.055 0.089 0.19 0.341 0.524 0.688 0.824 0.916  18 0.055 0.012 0.013 0.358 0.524 0.688 0.824 0.916  19 0.055 0.012 0.013 0.311 0.3 0.521 0.73 0.876 0.955	0.177	17	0.237	ĸ.	ω.	4.	•	. 63	•	. 78	0.846
bound  0.0544 0.277 0.362 0.455 0.55 0.642 0.726 0.8 0.  bound  0.0 0.05 0.221 0.32 0.431 0.543 0.648 0.741 0.818 0.  bound  0.0 0.05 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  0.133 0.208 0.303 0.415 0.535 0.653 0.776 0.848 0.  0.106 0.18 0.28 0.402 0.535 0.654 0.76 0.848 0.  0.106 0.18 0.28 0.405 0.535 0.654 0.76 0.848 0.  0.107 0.106 0.18 0.28 0.405 0.535 0.654 0.76 0.848 0.  0.108 0.209 0.303 0.415 0.535 0.654 0.76 0.848 0.  0.109 0.131 0.27 0.371 0.53 0.68 0.802 0.889 0.  0.001 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.  0.01 0.042 0.131 0.3 0.521 0.73 0.819 0.91 0.053 0.054 0.054 0.055 0.054 0.055 0.055 0.055 0.003 0	0.169	69	0.229	7	ω.	.46	•	. 63	•	. 78	0.85
bound  0.0 0.142 0.221 0.32 0.431 0.543 0.648 0.741 0.818 0.0  bound  0.0 0.54 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.0  0.133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.0  0.133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.0  0.133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.0  0.106 0.18 0.227 0.371 0.535 0.654 0.76 0.848 0.0  0.0054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.0  bound  0.0 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.0  10 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.0  10 0.054 0.119 0.327 0.371 0.53 0.68 0.802 0.889 0.0  10 0.055 0.123 0.227 0.371 0.524 0.703 0.843 0.911 0.0  11 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  12 0.015 0.015 0.121 0.341 0.524 0.688 0.824 0.916 0.0  13 0.035 0.089 0.19 0.341 0.524 0.688 0.824 0.916 0.0  14 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.0  15 0.015 0.015 0.123 0.225 0.363 0.524 0.688 0.824 0.916 0.0  16 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.0  17 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.955 0.0  18 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.955 0.0	0.1	12	0.203	7	ъ.	.45	•	.64	•	ω.	ω.
bound  0.0 .054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  0.0106 0.18 0.28 0.402 0.535 0.654 0.76 0.848 0.  0.0133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.  0.0106 0.18 0.28 0.402 0.535 0.654 0.76 0.848 0.  0.0106 0.19 0.227 0.415 0.535 0.663 0.776 0.848 0.  0.0054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  bound  0.0 0.54 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  10 0.053 0.089 0.19 0.341 0.524 0.703 0.843 0.916 0.  11 0.053 0.115 0.217 0.358 0.524 0.684 0.916 0.  12 0.053 0.089 0.19 0.341 0.524 0.684 0.916 0.  13 0.053 0.015 0.225 0.363 0.524 0.684 0.916 0.  14 0.053 0.015 0.217 0.358 0.524 0.684 0.916 0.915 0.001 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.	0.0	9	0.142		ų.	.43	•	.64	۲.	.81	ω.
bound </td <td></td>											
0.0 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  0.0106 0.18 0.228 0.402 0.535 0.663 0.776 0.864 0.  0.0106 0.18 0.28 0.402 0.535 0.654 0.76 0.848 0.  0.0133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.  0.0133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.  0.0133 0.208 0.303 0.415 0.535 0.654 0.76 0.848 0.  0.0106 0.18 0.28 0.402 0.535 0.663 0.776 0.848 0.  0.0054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  0.001 0.042 0.131 0.3 0.524 0.688 0.824 0.916 0.  0.0059 0.123 0.225 0.358 0.524 0.688 0.824 0.916 0.  0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.  0.0035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.  0.0035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.  0.001 0.042 0.131 0.3 0.524 0.688 0.824 0.916 0.  0.0059 0.109 0.341 0.524 0.688 0.824 0.916 0.  0.0059 0.109 0.341 0.524 0.703 0.843 0.931 0.		ō	nnd								
2         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.000           7         0.106         0.18         0.28         0.402         0.535         0.663         0.776         0.864         0.0           9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           9         0.134         0.228         0.402         0.535         0.663         0.776         0.848         0.0           10         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.0           1         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.889         0.889         0.899         0.999           1         0.035         0.042         0.131         0.3         0.524         0.703         0.874         0.916         0.916         0.916         0.931         0.916	05		•	.05	.10		.20				.40
2         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.0           7         0.106         0.18         0.28         0.402         0.535         0.663         0.776         0.864         0.0           9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           9         0.133         0.28         0.402         0.535         0.663         0.776         0.848         0.0           1         0.064         0.119         0.227         0.371         0.53         0.68         0.776         0.849         0.0           1         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.           1         0.054         0.119         0.371         0.53         0.52         0.38         0.843         0.916         0.           2         0.01         0.089         0.19         0.341         0.524 <td></td>											
7         0.106         0.18         0.28         0.402         0.535         0.663         0.776         0.864         0.0           9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           7         0.141         0.216         0.309         0.418         0.535         0.653         0.76         0.842         0.0           9         0.133         0.208         0.303         0.415         0.535         0.653         0.76         0.848         0.0           7         0.106         0.18         0.227         0.371         0.53         0.68         0.776         0.849         0.0           8         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.           9         0.01         0.042         0.13         0.53         0.68         0.876         0.955         0.           1         0.054         0.01         0.341         0.524         0.703         0.843         0.916         0.           1         0.053         0.115         0.217         0.354         0.524         0.688 <td>0.02</td> <td>22</td> <td>0.054</td> <td>۲.</td> <td>. 22</td> <td>ω.</td> <td>0.53</td> <td>•</td> <td>•</td> <td>. 88</td> <td>•</td>	0.02	22	0.054	۲.	. 22	ω.	0.53	•	•	. 88	•
9         0.133         0.208         0.303         0.415         0.535         0.654         0.76         0.848         0.0           7         0.141         0.216         0.309         0.418         0.535         0.654         0.76         0.842         0.0           9         0.133         0.208         0.303         0.415         0.535         0.663         0.76         0.848         0.0           7         0.106         0.18         0.227         0.371         0.53         0.68         0.76         0.848         0.0           9         0.054         0.119         0.227         0.371         0.53         0.68         0.802         0.889         0.           1         0.054         0.119         0.371         0.53         0.68         0.876         0.989         0.           1         0.035         0.089         0.19         0.341         0.524         0.703         0.843         0.916         0.           2         0.053         0.115         0.217         0.358         0.524         0.688         0.824         0.916         0.           1         0.053         0.115         0.217         0.358         0.524 </td <td>0.057</td> <td>27</td> <td>0.106</td> <td>٦.</td> <td>7</td> <td>4.</td> <td>53</td> <td>•</td> <td>•</td> <td>. 86</td> <td>•</td>	0.057	27	0.106	٦.	7	4.	53	•	•	. 86	•
7         0.141         0.216         0.309         0.418         0.535         0.654         0.755         0.842         0.0084         0.0133         0.208         0.303         0.415         0.535         0.663         0.776         0.848         0.0084         0.0084         0.0084         0.0084         0.0084         0.0084         0.0084         0.0084         0.0088         0.0089         0.0095         0.0095         0.0095         0.0095         0.0095         0.0095         0.0096         0.0	0.07	62	0.133	ς.	.30	4.	53	•	•	. 84	•
bound  bound  0.054 0.133 0.208 0.303 0.415 0.535 0.664 0.76 0.848 0.0  0.054 0.18 0.28 0.402 0.535 0.663 0.776 0.864 0.0  0.0054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.0  0.0 0.05 0.015 0.027 0.371 0.53 0.68 0.876 0.955 0.0  1 0.035 0.089 0.19 0.341 0.524 0.688 0.824 0.916 0.0  1 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  1 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  1 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  1 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  1 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0  1 0.053 0.019 0.341 0.524 0.088 0.824 0.916 0.0  2 0.01 0.042 0.131 0.3 0.521 0.73 0.843 0.931 0.0	0.08	37	0.141	ς.	.30	4.	53	•	•	. 84	•
0.106 0.18 0.28 0.402 0.535 0.663 0.776 0.864 0. 0.054 0.119 0.227 0.371 0.53 0.68 0.802 0.889 0.  ound 0.0 0.05 0.019 0.15 0.521 0.73 0.843 0.931 0. 0.053 0.015 0.25 0.363 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.25 0.363 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.053 0.015 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.055 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0. 0.01 0.042 0.131 0.3 0.521 0.73 0.843 0.931 0.	0.079	6/	0.133	7	.30	4.	53	•	92.0	. 84	•
0.054       0.119       0.227       0.371       0.53       0.68       0.802       0.889       0.0         0ound       0.0       .05       .10       .15       .20       .25       .30       .35       .35         0.01       0.042       0.131       0.3       0.524       0.703       0.843       0.955       0.         0.053       0.089       0.19       0.341       0.524       0.688       0.824       0.931       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.053       0.019       0.341       0.524       0.688       0.824       0.916       0.         0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.916       0.         0.035       0.042       0.131       0.3       0.524       0.703       0.843       0.916       0. <tr< td=""><td>0.0</td><td>2.2</td><td>0.106</td><td>٦.</td><td>7.</td><td>4.</td><td>53</td><td>99.</td><td>971.0</td><td>.86</td><td>•</td></tr<>	0.0	2.2	0.106	٦.	7.	4.	53	99.	971.0	.86	•
0.01       0.042       0.131       0.3       0.521       0.73       0.843       0.955       0.905         0.053       0.063       0.19       0.341       0.524       0.703       0.843       0.931       0.955       0.905         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.916       0.916       0.916       0.916       0.016       0.916       0.916       0.916       0.016 <td>٥.</td> <td>2</td> <td>0.054</td> <td>.11</td> <td>. 22</td> <td>ų.</td> <td>0.53</td> <td>9.</td> <td>.80</td> <td>. 88</td> <td>•</td>	٥.	2	0.054	.11	. 22	ų.	0.53	9.	.80	. 88	•
0.01       0.042       0.131       0.3       0.524       0.703       0.843       0.955       0.955         0.053       0.053       0.115       0.217       0.358       0.524       0.703       0.824       0.931       0.931         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.916         0.053       0.115       0.225       0.363       0.524       0.688       0.824       0.916       0.91         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.91         0.053       0.019       0.341       0.524       0.688       0.824       0.916       0.91         0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.916       0.91         0.01       0.042       0.131       0.3       0.521       0.73       0.843       0.955       0.955											
0.0       .05       .10       .15       .20       .25       .30       .35         0.01       0.042       0.131       0.3       0.524       0.73       0.843       0.955       0.9         0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.931       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.059       0.123       0.225       0.363       0.524       0.684       0.817       0.91       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.055       0.015       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.931       0.         0.01       0.042       0.131       0.3       0.521       0.73       0.875       0.955       0.		,	Ţ								
0.01     0.042     0.131     0.3     0.521     0.73     0.876     0.955     0.955       0.035     0.089     0.19     0.341     0.524     0.703     0.843     0.931     0.931       0.053     0.115     0.217     0.358     0.524     0.688     0.824     0.916     0.91       0.053     0.115     0.217     0.358     0.524     0.688     0.824     0.916     0.9       0.053     0.115     0.217     0.358     0.524     0.703     0.843     0.916     0.       0.035     0.089     0.19     0.341     0.524     0.703     0.843     0.931     0.       0.01     0.042     0.131     0.3     0.521     0.73     0.876     0.955     0.	.0	5	0	. 05	.10				.30		.40
0.01         0.042         0.131         0.3         0.521         0.73         0.876         0.955         0.9           0.035         0.089         0.19         0.341         0.524         0.703         0.843         0.931         0.9           0.053         0.115         0.217         0.358         0.524         0.684         0.824         0.916         0.           0.053         0.115         0.217         0.358         0.524         0.684         0.817         0.91         0.           0.053         0.115         0.217         0.358         0.524         0.688         0.824         0.916         0.           0.035         0.089         0.19         0.341         0.524         0.703         0.843         0.916         0.           0.035         0.042         0.131         0.3         0.521         0.73         0.876         0.955         0.											
0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.931       0.         0.053       0.115       0.217       0.358       0.524       0.684       0.817       0.916       0.         0.059       0.123       0.225       0.363       0.524       0.684       0.817       0.91       0.         0.053       0.115       0.217       0.358       0.524       0.688       0.824       0.916       0.         0.035       0.089       0.19       0.341       0.524       0.703       0.843       0.916       0.         0.01       0.042       0.131       0.3       0.521       0.73       0.876       0.955       0.	0.002	02	0.01	0.042	٦.		. 52	۲.	.87	σ.	•
0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0059 0.123 0.225 0.363 0.524 0.684 0.817 0.91 0.0053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.0000.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.001 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.000	0.0	11	0.035	0.089	۲.	. 34	. 52	۲.	. 84	ο.	•
0.059 0.123 0.225 0.363 0.524 0.684 0.817 0.91 0. 0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0. 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0. 0.01 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.	0.0	21	0.053	0.115	7	.35	. 52	9.	. 82	ο.	•
0.053 0.115 0.217 0.358 0.524 0.688 0.824 0.916 0.9 0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.9 0.01 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.9	0.0	25	0.059	0.123	ς.	. 36	. 52	٠.	.81	ο.	•
0.035 0.089 0.19 0.341 0.524 0.703 0.843 0.931 0.97 0.01 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.98	0.	121	0.053	0.115	7	.35	. 52	.68	. 82	ο.	ο.
0.01 0.042 0.131 0.3 0.521 0.73 0.876 0.955 0.98	0.0	111	0.035	0.089	٦.	.34	. 52	. 70	. 84	ο.	. 97
	0.0	02	0.01	0.042	٦.	•	. 52	۲.	. 87	ο,	. 98



Decreasing Percentile Rank,  $G_1^{\star}(\tau_1)$  – .10 =  $G_2^{\star}(\tau_2)$ . Also, the setting in Table 2 can be turned around to examine a student with a "real" decline from year-1 to year-2, in the sense of  $G_1^{\star}(\tau_1) > G_2^{\star}(\tau_2)$ . Table 4 shows values of y1y2 in (3) for a student with  $G_1^{\star}(\tau_1)$  – .10 =  $G_2^{\star}(\tau_2)$ . The entries in Table 4 also use the simplification of (3) in setting the year-1 and year-2 test reliability coefficients to be equal. Table 4 shows that even with  $G_1^{\star}(\tau_1)$  – .10 =  $G_2^{\star}(\tau_2)$ , the probability of obtaining an increase of 10 or more points in observed percentile rank is a large as .132 for test reliability .90.

### Insert Table 4 here

Guaranteeing positive improvement? Not possible, but it is of interest to ask, How much real improvement is needed in order to obtain high probability of an observed improvement? The entries in Table 5 set high probability as .90 and ask how large  $k = G_2^*(\tau_2) - G_1^*(\tau_1)$  needs to be in order for  $Pr\{G_2(S_2) - G_1(S_1) > 0\} = .90$ . Even for test reliability .95, k needs to be as large as .165.

Insert Table 5 here



Table 4.  $\Pr\{ \, G_2(S_2) - G_1(S_1) \leq \text{bound} \mid G_1^{\star}(\tau_1), \ G_2^{\star}(\tau_2) \} \ \text{for reliability .80 to .95 and} \\ G_1^{\star}(\tau_1) - .10 = \ G_2^{\star}(\tau_2) \ .$ 

Reliab	ility .80								
		bo	ound .						
	30	25	20	15	10	05	0.0	.05	.10
$G_1^*(\tau_1)$									
.20	0.111	0.171	0.252	0.356	0.478	0.609	0.733	0.831	0.899
.30	0.145	0.208	0.286	0.376	0.474	0.576	0.673	0.759	0.83
.40	0.166	0.23	0.304	0.387	0.474	0.563	0.649	0.728	0.797
.50	0.177	0.24	0.312	0.391	0.475	0.559	0.64	0.715	0.782
.60	0.177	0.24	0.312	0.391	0.475	0.559	0.64	0.715	0.782
.70	0.166	0.23	0.304	0.387	0.474	0.563	0.649	0.728	0.797
.80	0.145	0.208	0.286	0.376	0.474	0.576	0.673	0.759	0.83
.90	0.111	0.171	0.252	0.356	0.478	0.609	0.733	0.831	0.899
Roliah	ility .90								
iteliab.	ility .50								
			und						
	30	25	20	15	10	05	0.0	.05	.10
$G_1^*(\tau_1)$									
.20	0.042	0.091	0.178	0.31	0.484	0.67	0.825	0.921	0.969
.30	0.074	0.135	0.226	0.344	0.482	0.623	0.75	0.848	0.917
.40	0.096	0.163	0.252	0.361	0.482	0.605	0.717	0.812	0.883
.50	0.108	0.175	0.263	0.368	0.483	0.598	0.705	0.796	0.868
.60	0.108	0.175	0.263	0.368	0.483	0.598	0.705	0.796	0.868
.70	0.096	0.163	0.252	0.361	0.482	0.605	0.717	0.812	0.883
.80	0.074	0.135	0.226	0.344	0.482	0.623	0.75	0.848	0.917
. 90	0.042	0.091	0.178	0.31	0.484	0.67	0.825	0.921	0.969
Roliah	ility .95								
iteliab.	iiity .50	ho	ound		-				
	30	25	20	15	10	05	0.0	.05	.10
$G_1^{\star}(\tau_1)$			•	•					
.20	0.007	0.03	0.098	0.248	0.489	0.744	0.912	0.979	0.996
.30	0.022	0.064	0.152	0.297	0.487	0.682	0.836	0.93	0.976
.40	0.036	0.089	0.183	0.32	0.488	0.657	0.798	0.897	0.955
.50	0.044	0.101	0.197	0.33	0.488	0.647	0.783	0.882	0.944
.60	0.044	0.101	0.197	0.33	0.488	0.647	0.783	0.882	0.944
.70	0.036	0.089	0.183	0.32	0.488	0.657	0.798	0.897	0.955
.80	0.022	0.064	0.152	0.297	0.487	0.682	0.836	0.93	0.976
.90	0.007	0.03	0.098	0.248	0.489	0.744	0.912	0.979	0.996



Table 5. Values of k such that Pr{  $G_2(S_2) - G_1(S_1) > 0 \mid G_1^*(\tau_1) + k = G_2^*(\tau_2)$ } = .90 for reliability .80 to .95.

			$G_1^{\star}(\tau_1)$		
	.25	.40	.50	.60	.75
rel					
.80	0.342	0.343	0.318	0.277	0.193
. 85	0.285	0.294	0.277	0.245	0.174
. 90	0.222	0.237	0.227	0.204	0.149
95	0 148	0.165	0.161	0.148	0.112



## 3. Consistency of Percentile Rank Scores Over Years

It depends what the meaning of "consistency" is. Another approach to describing accuracy of the percentile rank scores over successive years is to consider the setting in which  $G_i^*(\tau_i)$  is the same over two (or more years), e.g., .60 in year-1 and year-2. And then ask, given constant  $G_i^*(\tau_i)$  ( $G_1^*(\tau_1) = G_2^*(\tau_2)$ ), how consistent are the observed  $G_1(S_1)$  and  $G_2(S_2)$ ?

In Rogosa (1999a), one approach to describing the accuracy of a percentile rank score was to calculate, for a student whose percentile rank under perfect measurement is  $100 \, \text{G}_{i}^{\star}(\tau_{i})$ :

 $\text{hit-rate}_i = \Pr\{ | G_i(S_i) - G_i^{\star}(\tau_i) | \leq \text{tolerance} | G_i^{\star}(\tau_i) \}.$ 

And from Rogosa (1999a), for year i

hit-rate<sub>i</sub> = 
$$\Phi[\{\Phi^{-1}[G_i^{\star}(\tau_i) + \text{tol}] - (\sqrt{\text{rel}_i})\Phi^{-1}[G_i^{\star}(\tau_i)]\}/(1 - \text{rel}_i)^{\frac{1}{2}}] - \Phi[\{\Phi^{-1}[G_i^{\star}(\tau_i) - \text{tol}] - (\sqrt{\text{rel}_i})\Phi^{-1}[G_i^{\star}(\tau_i)]\}/(1 - \text{rel}_i)^{\frac{1}{2}}].$$

And thus a measure of year-to-year consistency is the probability that  $G_1(S_1)$  is within the designated closeness to  $G_1^{\star}(\tau_1)$  and  $G_2(S_2)$  is within the designated closeness to  $G_2^{\star}(\tau_2)$  (with typically  $G_1^{\star}(\tau_1) = G_2^{\star}(\tau_2)$ ):

$$consistency_{12} = hit-rate_1 \cdot hit-rate_2$$
.

Table 6 presents values of consistency<sub>12</sub> for three values of the tolerance: tol = .01, tol=.025, and tol = .05. For example, with a test reliability of .90 for both years, a student "really" at the  $60^{th}$  percentile has probability .026 of both observed percentile rank scores being within 2.5 percentile points of the  $60^{th}$  percentile (i.e., in the range 57.5 to 62.5) and probability .101 of both years' observed percentile rank scores being within 5 percentile points of the  $60^{th}$  percentile.

Insert Table 6 here



Table 6. Year-1, Year-2 Consistency of Observed Percentile Rank Scores.

tolerance	.01					
			$G_1^*(\tau_1) = G$	$\tau_2^*(\tau_2)$		
	.25	.40	.50	.60	. 75	.90
rel						
0.8	0.003	0.002	0.002	0.002	0.003	0.009
0.825	0.004	0.002	0.002	0.002	0.004	0.011
0.85	0.004	0.003	0.003	0.003	0.004	0.013
0.875	0.005	0.003	0.003	0.003	0.005	0.016
0.9	0.006	0.004	0.004	0.004	0.006	0.02
0.925	0.008	0.006	0.005	0.006	0.008	0.026
0.95	0.012	0.008	0.008	0.008	0.012	0.04
tolerance	.025					
			$G_1^*(\tau_1) = G$	$_{2}^{*}(\tau_{2})$		
	.25	.40	. 50	.60	. 75	.90
rel						
0.8	0.019	0.013	0.012	0.013	0.019	0.058
0.825	0.022	0.015	0.014	0.015	0.022	0.067
0.85	0.026	0.018	0.017	0.018	0.026	0.078
0.875	0.031	0.021	0.02	0.021	0.031	0.094
0.9	0.038	0.026	0.025	0.026	0.038	0.117
0.925	0.051	0.035	0.033	0.035	0.051	0.155
0.95	0.075	0.052	0.049	0.052	0.075	0.224
tolerance	.05		*	.*		
			$G_1^{\star}(\tau_1) = G$	$t_2(\tau_2)$		
	.25	.40	.50	.60	.75	.90
rel						
0.8	0.075	0.052	0.049	0.052	0.075	0.216
0.825	0.085	0.059	0.056	0.059	0.085	0.245
0.85	0.099	0.069	0.065	0.069	0.099	0.281
0.875	0.118	0.082	0.077	0.082	0.118	0.329
0.9	0.145	0.101	0.095	0.101	0.145	0.393
0.925	0.189	0.133	0.125	0.133	0.189	0.486
0.95	0.269	0.192	0.181	0.192	0.269	0.625



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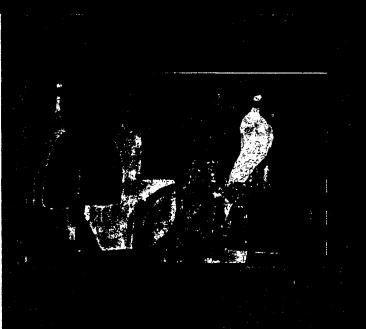
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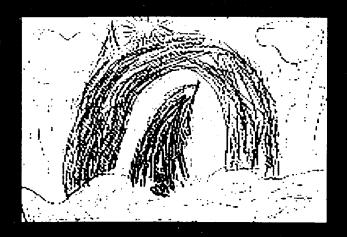
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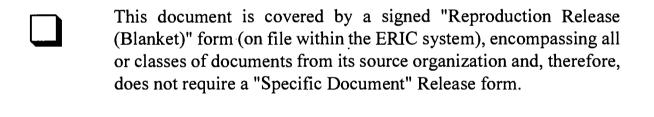
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